* R N - 8 1 6 5 / 7 0 0 *

RN-8165

B. E. II (Sem. IV) (All) Examination

May / June – 2010

Mathematics-IV

Time : Hours] [Total Marks :

Instruction :

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.

Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ **B. E. 2 (Sem. 4) (All)**

Name of the Subject :

☛ **Mathematics-4**

☛ Subject Code No. : **8 1 6 5** ☛ Section No. (1, 2,.....) : **1&2**

Seat No. :

Student's Signature

Section-I

Q-1 a) Do as directed. **[10]**

1. Define singular point along with state the singular point of $Logz$.
2. Write the principal part of the function $f(z)=\frac{1-\cos z}{z^2}$ and determine whether that point is a pole, a removable singular point, or an essential singular point.
3. Define argument of a complex number and hence determine principle argument of $z=\frac{-i}{-2-2i}$.
4. Reduce this quantity $\frac{1+2i}{3-4i}+\frac{2+i}{5i}$ to a real no.
5. Identify and sketch the set of points satisfying $|z-1|^2+|z+1|^2<8$.

b) Attempt the following. **[10]**

1. For a fixed $a\in C$, show that $\frac{|z-a|}{|1-\bar{a}z|}=1\ if\ |z|=1\ and\ 1-\bar{a}z\neq 0$.
2. Show that $u(x,y)=2x(1-y)$ is harmonic in some domain and find a harmonic conjugate $v(x,y)$.

Q-2 a) Show that $|\text{Re}(z)|\le|z|$ and $|\text{Im}(z)|\le|z|$. Also Show that $|z+w|^2=|z|^2+|w|^2+2\,\text{Re}(z\bar{w})$. Use this to prove triangle inequality $|z+w|\le|z|+|w|$. **[05]**

b) **Attempt any three of the following.** **[12]**

1. Show that $\left|\operatorname{Re}\left(2+\bar{z}+z^4\right)\right| \le 4$ when $|z| \le 1$.

2. Find the radius of convergence for the series $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!} z^n$.

3. Evaluate $\int_C \bar{z}.dz$. Where C is the right-half of the circle $|z| = 2$. Hence show that $\int_C \frac{dz}{z} = \pi i$.

4. Find the Laurent series that represents the function $f(z) = z^2 \sin\left(\frac{1}{z^2}\right)$

Q-3 a) Using residue theory, show that $\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)(x^2+4)}$. **[05]**

b) **Attempt any three of the following.** **[09]**

1. Use Cauchy's integral formula to evaluate $\left|\oint_C \frac{e^{2z}}{(z+1)} 4\,dz\right.$; where C is the circle $|z| = 3$.

2. Evalaute $\oint_C z^n dz$; Where C is the counter $|z| = 1$

3. Using Cauchy residue theory, evaluate $\int_C z^2 \sin\left(\frac{1}{2}\right) dz$.

4. Find all the zeros of the polynomial $z^5 + 1$.

Section-II

Q-4 a) Do as directed. **[10]**

1. Obtain second and third Backward difference of y_0.
2. Find the value of $\Delta \log f(x)$.
3. Define open-end method and state any two of it.
4. Define partial pivoting and complete pivoting.
5. State Runge-Kutta method of third and fourth order.

b) Attempt any three of the following **[12]**

1. State Newton's formula for Forward difference Interpolation and hence find the value of tan(0.12)
2. The velocity V of a particle at distance S from a point on its path is given by the following table

S (ft)	0	10	20	30	40	50	60
V(ft/s)	47	58	64	65	61	52	38

Estimate the time taken to travel 60 ft using Simpson's 1/3 and 3/8 rule.

3. State Newton's divided difference formula and find the value of f(8) and f(15), using following data.

X	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

4. Apply Trapezoidal and Simpson's rule to evaluate $\int_0^1 \sqrt{1-x^2}\, dx$.

Q-5 a) Solve any two of the following. **[08]**

1. Solve the following system of equations using partial pivoting by Gauss elimination method,

$$2x_1 + 2x_2 - 2x_3 = 8$$
$$-4x_1 - 2x_2 + 2x_3 = -14$$
$$-2x_1 + 3x_2 + 9x_3 = 9$$

2. Solve using modification of Gauss elimination method,

$$x + y + z = 7$$
$$3x + 3y + 4z = 24$$
$$2x + y + 3z = 16$$

3. Define fixed point, State the formula of fixed point Iteration method and using it find a real root of $x^3 - x - 1 = 0$ correct to three decimal.

b) Attempt any two of the following **[06]**

1. Discuss Euler method and hence find $y(1)$ given that,

$$\frac{dy}{dx} = x + y, y(0) = 1 \,\&\, h = 0.1$$

2. Use secant method to find approximate root of $x^3 = 2x + 1$ by taking $x_0 = 1.5$ & $x_1 = 2$.
3. Use Reglur-Falsi method to find root of the given equation correct to three decimals. $2x - \log_{10} x = 7$

Q-6 a) Attempt any two of the following **[08]**

1. Use Jacobi's method to solve

$4x + y + 3z = 17$

$x + 5y + z = 14$

$2x - y + 8z = 12$

2. Use Gauss-Seidel method to solve

$6x_1 + x_2 + x_3 = 6$

$x_1 + 8x_2 + 2x_3 = 4$

$3x_1 + 2x_2 + 10x_3 = -1$

3. Find the smallest Eigen value and corresponding eigen vectors of matrix $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

b) Attempt any two of following **[06]**

1. Use Taylor's series method to solve $\frac{dy}{dx} = 3x + y^2, y(0) = 1$. Also find $y(0.1)$.

2. Apply Euler's modified method to find y from x = 0 to 0.6, given that $\frac{dy}{dx} = 1 - 2xy, y(0) = 0 \,\&\, h = 0.2$

3. Use Runge-Kutta second order method to find the approximate value of y at x = 1.2 taking h = 0.1 where $\frac{dy}{dx} = x^2 + y^2 \,\&\, y(1) = 0$
